Philadelphia University
Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **01–04–2008**

There are 6 problems; you choose 4, no more no less.

1. Let R be a ring.
   (a) What is the meaning of a **subring** of R?
   (b) What is the meaning that R is a **field**?
   (c) The set $S = \{a + b\sqrt{5} \mid a,\, b \in Q\}$ is a subring of Q. Prove that S is a field.
2. Let R be a ring.
   (a) What is the meaning that R is an **integral domain**?
   (b) Prove that every field is an integral domain.
   (c) Prove that every finite integral domain is a field.
3. Let R and S be two rings.
   (a) What is the meaning of a **homomorphism** $\theta : R \to S$?
   (b) What is the meaning of an **isomorphism** $R \approx S$?
   (c) If $R \approx S$ and R is an integral domain, prove that S is also an integral domain.
4. Let $F[x]$ be a polynomial ring.
   (a) What is the meaning of an **ideal** of a ring?
   (b) What is the meaning of a **principal ideal**?
   (c) Prove that every ideal of $F[x]$ is principal.
5. Let f and $g \in F[x]$.
   (a) What is the meaning that f is **divisible** by g?
   (b) If $\alpha \in F$, prove that $f(x)$ is divisible by $x - \alpha$ if and only if $f(\alpha) = 0$.
   (c) If $F = Z_7$, show that $f(x) = x^3 - 3$ is not divisible by any polynomial of lower degree.
6. Let f and $g \in F[x]$.
   (a) What is the meaning of a **greatest common divisor** of f and g?
   (b) What is the meaning of **the** $\gcd(f, g)$?
   (c) If $F = Q$, evaluate $\gcd(x^5 + 4x,\, x^3 - x)$.

–Amin Witno