Philadelphia University

Department of Basic Sciences

Exam 2 **Abstract Algebra 2** **8–5–2007**

There are 6 problems, you choose 4, no more no less.

1. Let F be a field. We know that $F[x]$ is a ring. Prove that $F[x]$ is an integral domain, but not a field.

2. (a) What is the meaning of a principal ideal?
   (b) Prove that every ideal of $F[x]$ is principal.

3. Let $f, g \in \mathrm{Q}[x]$.
   (a) What is the definition of *the* $\gcd(f, g)$?
   (b) What is $\gcd(6x^4 + 2x^2 - 20, \quad 2x^3 - 2x^2 + 4x - 4)$?

4. Let $f \in \mathrm{Z}[x]$.
   (a) What is the meaning of a primitive polynomial?
   (b) Suppose that f is primitive. Prove that if f can be factored in $\mathrm{Q}[x]$ then it can be factored in $\mathrm{Z}[x]$.

5. (a) What is the meaning of an irreducible polynomial?
   (b) Prove that $x^3 - x + 1$ is reducible in $\mathrm{Z}_7[x]$ and factor it.
   (c) Is $7x^5 - 10x^3 + 14x^2 - 4x + 6$ irreducible in $\mathrm{Q}[x]$? Prove it.

6. (a) Prove that $f = x^3 + 2$ is irreducible in $\mathrm{Z}_7[x]$.
   (b) Prove that every element in the factor ring $\mathrm{Z}_7[x]/(f)$ is of the form $a+bx+cx^2$ where $a, b, c \in \mathrm{Z}_7$.
   (c) How many elements are in this factor ring?
   (d) What is $\mathrm{Z}_7[x]/(f)$ isomorphic to?

–Amin Witno