Final Exam **Abstract Algebra 1** **18/06/2023**

1. (1 point each)
   (a) Let $f = (1, 4, 2, 6)(3, 8)$ and $g = (1, 7, 4, 3, 2, 5, 8) \in S_8$. Find $g^{-1} \circ f$.
   (b) Find $F_3 \circ F_6$ in D_7.
   (c) Write all the cosets of A_3 in S_3.
   (d) Determine $[D_4 : S_4]$.
   (e) Let $G = \{(1, 3, 4)(2, 6), (1, 4, 3), (2, 6), (1, 3, 4), (1, 4, 3)(2, 6), e\}$. Find $S_G(6)$.
   (f) Let $G = \{(1, 3, 4)(2, 6), (1, 4, 3), (2, 6), (1, 3, 4), (1, 4, 3)(2, 6), e\}$. Find $O_G(4)$.
2. (1 point each) Let $f : \mathbb{Z}_{12} \to \mathbb{Z}_{10}$ such that $f(n) = 3n$.
   (a) What is the range of f?
   (b) Is f one-to-one? Why?
   (c) Is f onto? Why?
   (d) Find a counter-example to show f is not homomorphism.
3. (2 points each) True or False? And why?
   (a) Is $U_5 \approx U_{10}$? Why or why not?
   (b) Is $U_{10} \approx U_8$? Why or why not?
4. (6 points) Let $f : G \to H$ be a group homomorphism.
   (a) Prove that $\ker(f)$ is a subgroup of G.
   (b) Prove that $\ker(f)$ is normal.
5. (5 points) Let H and K be subgroups of G with $|H| = 15$ and $|K| = 28$. Prove that $H \cap K = \{e\}$.
6. (5 points) Let G be a group and $H = \{x^3 \mid x \in G\}$. Assume H is a subgroup. Prove that H is normal.
7. (5 points) Let G be a group with center $Z(G)$. Prove that if $xy \in Z(G)$, then $xy = yx$.
8. (5 points) Let $f : G \to G'$ be a group homomorphism with $\ker(f) = H$. Let $\mathcal{F} : G/H \to G'$ such that $\mathcal{F}(Ha) = f(a)$. Prove that $\mathcal{F}$ is a homomorphism and prove it is one-to-one.

–Amin Witno