Midterm Exam **Abstract Algebra 1** **09/05/2023**

1. (4 points) Short Answer.
   (a) Find $(3,4)^{-1}$ in the group $\mathbb{Z}_9 \times U_9$
   (b) Find $|3|$ in the group U_{11}
   (c) Determine $[\mathbb{Z}_{36} : H]$ where $H = \langle 15 \rangle$
   (d) Count how many generators of the group $\mathbb{Z}_{20}$
2. (5 points) Let $G = \{x \in \mathbb{R} \mid x \neq -2\}$ and $a \star b = ab + 2a + 2b + 2$ for all $a, b \in G$. Prove that G is a group.
3. (4 points) Let G be a group and $g \in G$. Let $H = \{z \in G \mid zg = gz\}$. Prove that H is a subgroup of G.
4. (3 points) Let $G = GL(2, \mathbb{Q})$. Prove that the subgroup $H = \{A \in G \mid \det A = 1\}$ is normal.
5. (4 points) Construct the Cayley table for the factor group G/H where $G = U_{13}$ and $H = \langle 3 \rangle$
6. (4 points) Prove that the group $\mathbb{Z}_9 \times U_{13}$ is not cyclic.
7. (3 points) Let G be a group such that $x^2 = e$ for all $x \in G$. Prove that G is abelian.
8. (3 points) Let G be a cyclic group. Prove that G is abelian.
9. (Bonus 3 points) Let G be a group and $a \in G$. Prove that $C(a) = C(a^{-1})$

–Amin Witno