



Philadelphia University
Department of Basic Sciences and Mathematics

First Exam

Abstract Algebra I (250342)

15/11/2016

Name: _____ **Number:** _____ **Section:** _____

Question 1: (5 points) Mark each of the following true or false (if false give an example):

- a) A group may have more than one identity element ().
- b) The empty set can be considered to be a group ().
- c) The associative law holds in every group ().
- d) Every group is a subgroup of itself ().
- e) Every set of numbers which is a group under addition is also a group under multiplication ().

Question 2: (3 points) Let G be a group with identity e . Prove that if $a^2 = e$ for all $a \in G$ then G is abelian.

Question 3: (5 points)

1. If $S = \{3^k | k \in \mathbb{Z}\}$. Prove that S is a subgroup of \mathbb{R}^* :

2. Prove or disprove the group G is cyclic for $G = U_4 \times \mathbb{Z}_3$?

Question 4: (5 points) Let G be the set of all numbers except -1 . Define a binary operation $*$ on G such that $a * b = a + b + ab$. Prove that G is a group.

Question 5: (2 points) Find a^{-1} for the group element $a \in G$ of the following:

1. $7 \in U_9$.

2. $(11, 2) \in U_{12} \times \mathbb{Z}_4$