PHILADELPHIA UNIVERSITY

DEPARTMENT OF BASIC SCIENCES

Exam 1   **Abstract Algebra 1**   **06–11–2012**

Part 1: Short Answer

1. What is the identity of the group $\mathbb{R}^*$?

2. What is an example of a group that is not abelian?

3. What is the inverse of $\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$ in the group $M(2, \mathbb{Z})$?

4. What are the elements of the group U_{18}?

5. What is the result of the operation $(2, 5) \star (8, 5)$ in the group $\mathbb{Z}_9 \times U_7$?

6. What is the inverse of 7 in the group U_9?

7. The group $\mathbb{Q}^*$ is a subgroup of $\mathbb{Q}$. True or false?

8. What is an example of a subgroup of $\mathbb{Z}$?

Part 2: Complete Solution

1. Let S be a set and $G = \{$all subsets of $S\}$. Define $A \star B = A \cup B$ for all $A, B \in G$. Prove that G is not a group with this operation.

2. Let G be a group such that $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$. Prove that G is abelian.

3. Let G be a group and $g \in G$. Let $H = \{x \in G \mid xg = gx\}$. Prove that H is a subgroup of G.

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