Philadelphia University

Department of Basic Sciences

Final Exam **Abstract Algebra 1** **26–01–2009**

Choose any 5 problems from the following 10 problems.

1. Let $G = \{x \in R \mid x \neq -1\}$. Prove that G is a group under the operation $\star$, where $a \star b = a + b + ab$.
2. Draw the subgroup lattice for the group U_{13}.
3. Let H be a subgroup of a group G. If $[G : H] = 2$, prove that H is normal.
4. Suppose that $\theta : G \to H$ is a group homomorphism. If $a \in G$, prove that $|\theta(a)|$ divides $|a|$.
5. Let N be a normal subgroup of G. Prove that the factor group G/N is abelian if and only if $aba^{-1}b^{-1} \in N$ for all $a, b \in G$.
6. Suppose that G is a group which is isomorphic to another group H. Show that G is cyclic if and only if H is cyclic.
7. Let G be a group of order 3. Show that $G \approx Z_{18}/\langle 3 \rangle$.
8. Draw the multiplication table for the group S_3. Is S_3 cyclic? Why or why not?
9. The subgroup $N = \langle (1, 3)(2, 4) \rangle$ is normal in D_4. Draw the multiplication table for the factor group D_4/N.
10. In the group D_n, show that the composition of a rotation with a reflection is a reflection.

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!
2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.
3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.
4. Do exactly five problems. No bonus points will be given to a sixth solution and beyond. If you have extra time, double check your work.

–Amin Witno