Philadelphia University

Department of Basic Sciences

Exam 2 **Abstract Algebra 1** **23–12–2008**

Choose any 4 problems from the following 8 problems.

1. The group U_{13} is cyclic. Draw its subgroup lattice.
2. Let N be a normal subgroup of a group G. If H is any subgroup of G, prove that $N \cap H$ is a normal subgroup of H.
3. Draw the multiplication table for the factor group G/N, where $G = U_{15}$ and the subgroup $N = \langle 4 \rangle$.
4. Let G be an abelian group. Prove that the map $\theta : G \to G$ given by $\theta(a) = a^{-1}$ is an isomorphism.
5. Suppose that $\theta : G \to H$ is a group homomorphism. Prove that $\ker(\theta)$ is a normal subgroup of G.
6. Suppose that G is a group which is isomorphic to another group H. Show that G is cyclic if and only if H is cyclic.
7. Let G and H be two groups of order 5. Prove that G is isomorphic to H.
8. Is $Z_2 \times Z_2$ isomorphic to Z_4? Prove true or false.

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!
2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.
3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.
4. Do exactly four problems. No bonus points will be given to a fifth solution and beyond. If you have extra time, double check your work.

–Amin Witno